Preprint typeset in JHEP style - HYPER VERSION

MPP-2004-173

arXiv:hep-th/0412250v2 25 Jan 2005

Supersymmetric AdS_4 compactifications of IIA supergravity

Dieter Lüst

Max-Planck-Institut für Physik –Theorie
Föhringer Ring 6, 80805 München, Germany
&
Department für Physik, Ludwig-Maximillians-Universität
Theresienstr. 37, 80333 München, Germany
E-mail: luest@mppmu.mpg.de, luest@theorie.physik.uni-muenchen.de

Dimitrios Tsimpis

Max-Planck-Institut für Physik –Theorie
Föhringer Ring 6, 80805 München, Germany
E-mail: tsimpis@mppmu.mpg.de

Abstract: We derive necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications of (massive) IIA supergravity to AdS_4 in the language of $SU(3)$ structures. We find new solutions characterized by constant dilaton and nonzero fluxes for all form fields. All fluxes are given in terms of the geometrical data of the internal compact space. The latter is constrained to belong to a special class of half-flat manifolds.

Keywords: Superstring vacua, supergravity models.

Contents

1. Introduction and Summary

Flux compactifications [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] are currently being studied intensively, not least for their potential phenomenological interest. Perhaps their most attractive feature is that

they allow the possibility of fixing (part of) the geometrical moduli of the internal space. Moreover fluxes generally source warp factors, which can provide a mechanism for generating hierarchies.

In many cases it suffices to work in an approximation where the back-reaction of the fluxes on the internal manifold is ignored. In $\mathcal{N} = 2$ type II compactifications to four-dimensional Minkowski space, for example, one continues to treat the internal manifold as if it were a Calabi-Yau, even after giving expectation values to the antisymmetric tensors along the internal directions. This situation is usually described as 'Calabi-Yau with fluxes' despite the fact that it does not correspond to a true supergravity solution.

This approach is motivated partly by the fact that the physics community has grown particularly fond of Calabi-Yau manifolds, on which one can use familiar tools from algebraic geometry. Once fluxes are turned on, however, the internal manifold is deformed away from the Calabi-Yau point and, generically, it will even cease to be complex. The manifolds which appear naturally in the setup of the present paper, for example, belong to the class of *half-flat* manifolds [15] also known as *half-integrable* [16] about which little is known. Notably, they appear in the mirror-symmetric picture of 'Calabi-Yau with fluxes' compactifications [17, 18].

For many practical purposes it has proven fruitful to work within the approximation described above and to ignore the back-reaction of the fluxes. Nevertheless it would still be desirable to obtain exact results, corresponding to genuine supergravity solutions. Ideally, one would like to be able to classify and systematically construct concrete examples of internal manifolds satisfying the requirements for a consistent (supersymmetric) vacuum with fluxes. This task, however, is well beyond our current technology. The knowledge of exact solutions may provide clues as to how to set up some kind of perturbative expansion for which the order parameter would be the flux. 'Calabi-Yau with fluxes' would then correspond to the leading-order term in this expansion.

The subject of supersymmetric supergravity compactifications is of course not a new one. More recently, it has become clear that the most suitable language for the description of flux compactifications is that of G-structures. There is already a considerable amount of literature on the subject, see [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] for an inexhaustive list of references.

The necessary conditions for $\mathcal{N} = 1$ IIA (and IIB) supergravity compactifications to four-dimensional Minkowski space were given in the language of $SU(3)$ structures in [35, 36]. The authors of these references examined the conditions for unbroken supersymmetry but did not impose the Bianchi identities or the equations of motion. The latter were taken into account in reference [29]. A class of $\mathcal{N} = 1$ compactifications of Romans' supergravity to AdS_4 was given in [37, 38], corresponding to the internal manifold being *nearly-Kähler*. In the massless limit these solutions reduce to four-dimensional Minkowski space times a Calabi-Yau, and all fluxes vanish.

In the present paper we derive the necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications of (massive) IIA supergravity to AdS_4 in the context of $SU(3)$ structures, and we find a new class of solutions. These are characterized by constant dilaton and scalar fluxes (the term 'scalar' referring to the decomposition in terms of the $SU(3)$ structure group) turned on for all form fields. In addition, the (massive) two-form can have a nonzero primitive piece. The intrinsic torsion of the internal manifold

X_6 is constrained by supersymmetry and the Bianchi identities to lie in

$$\tau \in \mathcal{W}_1^- \oplus \mathcal{W}_2^- \tag{1.1}$$

and therefore X_6 is a half-flat manifold. Recall that the latter is a manifold whose intrinsic torsion is contained in $\mathcal{W}_1^- \oplus \mathcal{W}_2^- \oplus \mathcal{W}_3$. Within the class of half-flat manifolds, our solutions are 'orthogonal' to the example based on the Iwasawa manifold encountered in [21], in the sense that for the latter the intrinsic torsion is entirely contained in $\mathcal{W}_3$. In addition to (1.1), the Bianchi identities require the exterior derivative of $\mathcal{W}_2^-$ to be proportional to the real part of the $(3,0)$ form on the manifold X_6,

$$d\mathcal{W}_2^- \propto Re(\Omega) \ . \tag{1.2}$$

Moreover, all form field components are expressible in terms of the geometrical data of X_6. The equations-of-motion are then satisfied with no further requirements.

In contrast to [37, 38] our solutions reduce in the massless limit to AdS_4 times a six-dimensional manifod X_6 of the type (1.1, 1.2). The lift to M-theory can then be taken, and leads to a seven-dimensional internal manifold which is a twisted circle fibration over X_6. This theory is expected to admit a three-dimensional conformal field theory dual, see [39] for a recent discussion and [40] for earlier work on this subject.

In section 5 we examine examples of six-dimensional manifolds with properties (1.1,1.2). More specifically, in section 5.1 we construct a class of examples of six-dimensional manifolds which are T^2 fibrations over K3. In addition, in section 5.2 we examine a one-parameter family of examples based on the Iwasawa manifold. To our knowledge, of all the nilmanifolds considered in the mathematical literature [41, 15, 16, 42], this is the only one for which (1.1) holds. This example turns out to be a degeneration of the case considered in section 5.1, whereby the $K3$ base is replaced by a T^4. Although the examples of section 5 do satisfy equations (1.1,1.2), they fail to reproduce the specific constant of proportionality between $d\mathcal{W}_2^-$ and the real part of the $(3,0)$ form required by the Bianchi identities. The construction of more examples of the type (1.1,1.2) will have to await further developments in the mathematical literature.

In section 2 we review Romans' ten-dimensional supergravity and examine the integrability of the supersymmetry variations. We conclude that imposing supersymmetry, the Bianchi identities and the form equations of motion suffices for the dilaton and Einstein equations to be automatically satisfied. Similar statements have previously appeared in the literature (see [22],[43] for a discussion in the context of eleven-dimensional supergravity) but, to our knowledge, not in the context of the present paper. In section 3 we reduce on $AdS_4 \times X_6$ taking into account the fact that the internal manifold possesses an $SU(3)$ structure. The necessary and sufficient conditions for $\mathcal{N}=1$ supersymmetry are derived in section 4, where our solutions are presented. Section 5 is devoted to the explicit construction of examples. The two appendices contain our conventions and some useful technical results.

2. Massive IIA

This section contains a review of Romans massive supergravity [44]. It is included here in order to establish notation and conventions.

The bosonic part of the action reads

$$\mathcal{L} = \int \{R * 1 - \frac{1}{2}d\phi \wedge *d\phi - \frac{1}{2}e^{\phi/2}G \wedge *G - \frac{1}{2}e^{-\phi}H \wedge *H - 2m^2 e^{3\phi/2}B' \wedge *B' \\ + \frac{1}{2}(dC')^2 \wedge B' + \frac{m}{3}dC' \wedge (B')^3 + \frac{m^2}{10}(B')^5 - 2m^2 e^{5\phi/2} * 1\} \ , \tag{2.1}$$

where

$$H = dB' \tag{2.2}$$

and

$$G = dC' + m(B')^2 \ . \tag{2.3}$$

These forms obey the Bianchi identities

$$\begin{aligned} dH &= 0 \\ dG &= 2mB' \wedge H \ . \end{aligned} \tag{2.4}$$

Note that we are using 'superspace' conventions for the forms:

$$\begin{aligned} A_{(n)} &= \frac{1}{n!}dx^{M_n} \wedge \ldots dx^{M_1} A_{M_1 \ldots M_n} \\ d(A_{(n)} \wedge B_{(q)}) &= A_{(n)} \wedge dB_{(q)} + (-)^q dA_{(n)} \wedge B_{(q)} \ . \end{aligned} \tag{2.5}$$

Recall that there is no (known) covariant lift of massive IIA to eleven dimensions. To make contact with the massless IIA supergravity of [45, 46, 47], one introduces a Stückelberg gauge potential A so that

$$\begin{aligned} mB' &= mB + \frac{1}{2}F \\ mC' &= mC - \frac{1}{4}A \wedge F \\ F = dA; \qquad H &= dB; \qquad G = dC + B \wedge F + mB^2 \ . \end{aligned} \tag{2.6}$$

The Bianchi identities of the forms read

$$\begin{aligned} dF &= 0 \\ dH &= 0 \\ dG &= H \wedge F + 2mB \wedge H \ . \end{aligned} \tag{2.7}$$

After introducing the Stückelberg field, the theory is invariant under

$$\begin{aligned} A &\to A + m\Lambda \\ B &\to B - \frac{1}{2}d\Lambda \\ C &\to C + \frac{1}{2}A \wedge d\Lambda + \frac{m}{4}\Lambda \wedge d\Lambda \ . \end{aligned} \tag{2.8}$$

Moreover, one can check that in terms of the fields A, B, C the Chern-Simons terms in (2.1) can be rewritten, up to a total derivative, as

$$CS = \frac{1}{2}dC^2 \wedge B + \frac{1}{2}dC \wedge B^2 \wedge F + \frac{1}{6}B^3 \wedge F^2 + \frac{m}{3}dC \wedge B^3 + \frac{m}{4}B^4 \wedge F + \frac{m^2}{10}B^5 \ ,$$

so that the $m \to 0$ limit can now be taken and the Lagrangian reduces to the one of massless IIA supergravity.

Equations of motion

The equations of motion that follow from (2.1) are

$$\begin{aligned}
0 &= R_{MN} - \frac{1}{2}\nabla_M\phi\nabla_N\phi - \frac{1}{12}e^{\phi/2}G_{MPQR}G_N{}^{PQR} + \frac{1}{128}e^{\phi/2}g_{MN}G^2 \\
&\quad - \frac{1}{4}e^{-\phi}H_{MPQ}H_N{}^{PQ} + \frac{1}{48}e^{-\phi}g_{MN}H^2 \\
&\quad - 2m^2e^{3\phi/2}B'_{MP}B'_N{}^P + \frac{m^2}{8}e^{3\phi/2}g_{MN}(B')^2 - \frac{m^2}{4}e^{5\phi/2}g_{MN} \qquad (2.9)\\
0 &= \nabla^2\phi - \frac{1}{96}e^{\phi/2}G^2 + \frac{1}{12}e^{-\phi}H^2 - \frac{3m^2}{2}e^{3\phi/2}(B')^2 - 5m^2e^{5\phi/2} \qquad (2.10)\\
0 &= d(e^{-\phi} * H) - \frac{1}{2}G\wedge G + 2m\ e^{\phi/2}B'\wedge *G + 4m^2e^{3\phi/2} * B' \qquad (2.11)\\
0 &= d(e^{\phi/2} * G) - H\wedge G\ . \qquad (2.12)
\end{aligned}$$

Note that the integrability condition following from (2.11),

$$0 = e^{\phi/2}H\wedge *G + 2m\ d(e^{3\phi/2} * B')\ , \qquad (2.13)$$

becomes in the massless limit the equation of motion for the gauge field.

Supersymmetry

The gravitino and dilatino supersymmetry variations read

$$\delta\Psi_M = \mathcal{D}_M\epsilon \qquad (2.14)$$

and

$$\begin{aligned}
\delta\lambda = \{&-\frac{1}{2}\Gamma^M\nabla_M\phi - \frac{5m\ e^{5\phi/4}}{4} + \frac{3m\ e^{3\phi/4}}{8}B'_{MN}\Gamma^{MN}\Gamma_{11} \\
&+ \frac{e^{-\phi/2}}{24}H_{MNP}\Gamma^{MNP}\Gamma_{11} - \frac{e^{\phi/4}}{192}G_{MNPQ}\Gamma^{MNPQ}\}\epsilon\ , \qquad (2.15)
\end{aligned}$$

where the supercovariant derivative is given by

$$\begin{aligned}
\mathcal{D}_M := \nabla_M &- \frac{m\ e^{5\phi/4}}{16}\Gamma_M - \frac{m\ e^{3\phi/4}}{32}B'_{NP}(\Gamma_M{}^{NP} - 14\delta_M{}^N\Gamma^P)\Gamma_{11} \\
&+ \frac{e^{-\phi/2}}{96}H_{NPQ}(\Gamma_M{}^{NPQ} - 9\delta_M{}^N\Gamma^{PQ})\Gamma_{11} + \frac{e^{\phi/4}}{256}G_{NPQR}(\Gamma_M{}^{NPQR} - \frac{20}{3}\delta_M{}^N\Gamma^{PQR}) \qquad (2.16)
\end{aligned}$$

and ϵ is the susy parameter. One can transform to the string frame by rescaling $e_A{}^M \to e^{\phi/4}e_A{}^M$.

Integrability

We will now argue that in a purely bosonic supersymmetric background the vanishing of the supersymmetric variations of the fermions together with the Bianchi identities and equations-of-motion for the forms imply (under a further mild assumption which is satisfied by the compactifications considered

in the present paper –see below) the dilaton and Einstein equations. To our knowledge, this is the first time this has been shown in the context of the present paper. For the purposes of this subsection we set $B'=0$ for simplicity of presentation. The conclusion does not change by introducing a nonzero B'-field. We have found [48] extremely useful in the following computation.

In a bosonic supersymmetric background, the supersymmetric variations of the fermions have to vanish. Assuming this to be the case, one can use the gravitino variation to obtain an expression for the commutator of two supercovariant derivatives acting on the susy parameter,

$$2\mathcal{D}_{[M}\mathcal{D}_{N]}\epsilon = \{\frac{1}{4}R_{MNPQ}\Gamma^{PQ} + \dots\}\epsilon = 0 \ . \tag{2.17}$$

Furthermore, the vanishing of the dilatino variation gives

$$\frac{5m}{4}e^{5\phi/4}\epsilon = \{-\frac{1}{2}\Gamma^{M}\nabla_{M}\phi - \frac{e^{\phi/4}}{192}G_{MNPQ}\Gamma^{MNPQ}\}\epsilon \ . \tag{2.18}$$

Taking the 'square' of the above expression we get,

$$\frac{1}{4}(\nabla\phi)^2\epsilon = \{\frac{25m^2}{16}e^{5\phi/2} + \dots\}\epsilon \ . \tag{2.19}$$

Multiplying (2.17) by Γ^N and substituting (2.18,2.19) we obtain

$$\begin{aligned}0 = \{ &-\frac{1}{2}\Gamma^{N}(R_{MN} - \frac{1}{2}\nabla_{M}\phi\nabla_{N}\phi - \frac{1}{12}e^{\phi/2}G_{MPQR}G_{N}{}^{PQR} + \frac{1}{128}e^{\phi/2}G^2 g_{MN} - \frac{m^2}{4}e^{5\phi/2}g_{MN})\\ &+ \frac{e^{\phi/4}}{256}\Gamma_{M}{}^{IJKLP}\nabla_{[I}G_{JKLP]} - \frac{25e^{\phi/4}}{768}\Gamma^{IJKL}\nabla_{[M}G_{IJKL]}\\ &+ \frac{e^{-\phi/4}}{64}\Gamma_{M}{}^{IJK}\nabla^{L}(e^{\phi/2}G_{LIJK}) - \frac{5e^{-\phi/4}}{64}\Gamma^{IJ}\nabla^{L}(e^{\phi/2}G_{LMIJ})\\ &+ \frac{e^{\phi/2}}{4608}(\Gamma_{MI_1\dots I_8}G^{I_1\dots I_4}G^{I_5\dots I_8} - \frac{1}{4}\Gamma_{M}\Gamma_{I_1\dots I_8}G^{I_1\dots I_4}G^{I_5\dots I_8}) \}\epsilon \ .\end{aligned} \tag{2.20}$$

Imposing the Bianchi identities and equations of motion for the forms, equation (2.20) takes the form $E_{MN}\Gamma^{N}\epsilon = 0$, where $E_{MN}=0$ are the Einstein equations. Multiplying this by $E_{MP}\Gamma^{P}$ implies $E_{MN}E_{M}{}^{N} = 0$ (no summation over M). Furthermore, if E_{M0} vanishes for $M \neq 0$, the remaining terms in the sum are positive-definite and one obtains $E_{MN}=0$, $M\neq 0$. Finally, $E_{0N}E_{0}{}^{N}=0$ implies $E_{00}=0$ and therefore $E_{MN}=0$, for all M,N. A similar statement can be made for the dilaton equation, as one can see by acting on the dilatino variation with $\Gamma^{N}\nabla_{N}$.

In conclusion, supersymmetry together with the Bianchi identities and equations of motion for the forms imply that the dilaton and Einstein equations are satisfied, provided $E_{0M}=0$ for $M\neq 0$.

3. $M_{1,3}\times X_6$ backgrounds

3.1 Supersymmetry

Let us now assume that spacetime is of the form of a warped product $M_{1,3}\times_{\omega} X_6$, where $M_{1,3}$ is AdS_4 (or $\mathbb{R}^{1,3}$) and X_6 is a compact manifold. The ten dimensional metric reads

$$g_{MN}(x,y) = \begin{pmatrix} \Delta^2(y)\hat{g}_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(y) \end{pmatrix} \ , \tag{3.1}$$

where x is a coordinate on $M_{1,3}$ and y is a coordinate on X_6. We will also assume that the forms have nonzero y-dependent components along the internal directions, except for the four-form which will be allowed to have an additional component proportional to the volume of $M_{1,3}$

$$G_{\mu\nu\kappa\lambda} = \sqrt{g_4} f(y) \varepsilon_{\mu\nu\kappa\lambda} \ , \tag{3.2}$$

where f is a scalar on X_6. Note that with these assumptions the $E_{M0} = 0$ for $M \neq 0$ condition is satisfied, and therefore we need only check supersymmetry the Bianchi identities and the equations of motion for the forms.

The requirement of $\mathcal{N} = 1$ supersymmetry in $4d$ (4 real supercharges) implies the existence of a globally defined complex spinor η on X_6. As a consequence the structure group of X_6 reduces to $SU(3)$, as explained in the following subsection in more detail. In addition, on $M_{1,3}$ there is a pair of Weyl spinors (related by complex conjugation), each of which satisfies the Killing equation

$$\hat{\nabla}_\mu \theta_+ = W \hat{\gamma}_\mu \theta_- \ ; \qquad \hat{\nabla}_\mu \theta_- = W^* \hat{\gamma}_\mu \theta_+ \ , \tag{3.3}$$

where hatted quantities are computed using the metric $\hat{g}_{\mu\nu}$, and the complex constant W is related to the scalar curvature $\hat{R}$ of $M_{1,3}$ through $\hat{R} = -24|W|^2$. Spinor conventions are given in appendix A.

The ten-dimensional spinor $\epsilon = \epsilon_+ + \epsilon_-$ decomposes as

$$\epsilon = (\alpha\theta_+ \otimes \eta_+ - \alpha^*\theta_- \otimes \eta_-) + (\beta\theta_+ \otimes \eta_- - \beta^*\theta_- \otimes \eta_+) \ , \tag{3.4}$$

where α, β are complex functions on X_6, undetermined at this stage. The spinor ϵ thus defined is Majorana.

Substituting these Ansätze in the supersymmetry transformations we obtain

$$\begin{aligned} 0 =\ & \alpha\nabla_m\eta_+ + \partial_m\alpha\eta_+ + \alpha\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_+ - \beta\frac{me^{5\phi/4}}{16}\gamma_m\eta_- + 3i\beta f\frac{e^{\phi/4}}{32}\gamma_m\eta_- \\ & + \beta\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np} - 14\delta_m{}^n\gamma^p)\eta_- + \beta\frac{e^{\phi/4}}{256}G_{npqr}(\gamma_m{}^{npqr} - \frac{20}{3}\delta_m{}^n\gamma^{pqr})\eta_- \end{aligned} \tag{3.5}$$

$$\begin{aligned} 0 =\ & \beta^*\nabla_m\eta_+ + \partial_m\beta^*\eta_+ - \beta^*\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_+ + \alpha^*\frac{me^{5\phi/4}}{16}\gamma_m\eta_- + 3i\alpha^* f\frac{e^{\phi/4}}{32}\gamma_m\eta_- \\ & + \alpha^*\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np} - 14\delta_m{}^n\gamma^p)\eta_- - \alpha^*\frac{e^{\phi/4}}{256}G_{npqr}(\gamma_m{}^{npqr} - \frac{20}{3}\delta_m{}^n\gamma^{pqr})\eta_- \ , \end{aligned} \tag{3.6}$$

from the 'internal' components of the gravitino variation and

$$\begin{aligned} 0 =\ & \alpha\Delta^{-1}W\eta_+ + \beta^*\frac{me^{5\phi/4}}{16}\eta_+ - 5i\beta^* f\frac{e^{\phi/4}}{32}\eta_+ - \beta^*\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_+ \\ & + \alpha^*\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_- - \beta^*\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_+ - \frac{1}{2}\alpha^*\partial_m(ln\Delta)\gamma^m\eta_- \end{aligned} \tag{3.7}$$

$$\begin{aligned} 0 =\ & \beta^*\Delta^{-1}W^*\eta_+ + \alpha\frac{me^{5\phi/4}}{16}\eta_+ + 5i\alpha f\frac{e^{\phi/4}}{32}\eta_+ + \alpha\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_+ \\ & + \beta\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_- - \alpha\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_+ + \frac{1}{2}\beta\partial_m(ln\Delta)\gamma^m\eta_- \ , \end{aligned} \tag{3.8}$$

from the noncompact piece. Note that these equations are complex. Similarly from the dilatino we obtain

$$
\begin{aligned}
0 &= \frac{1}{2}\alpha^*\partial_m\phi\gamma^m\eta_- - \alpha^*\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_- - \beta^*\frac{5me^{5\phi/4}}{4}\eta_+ \\
&+ i\beta^* f\frac{e^{\phi/4}}{8}\eta_+ - \beta^*\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_+ - \beta^*\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_+ \qquad (3.9) \\
0 &= \frac{1}{2}\beta\partial_m\phi\gamma^m\eta_- + \beta\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_- + \alpha\frac{5me^{5\phi/4}}{4}\eta_+ \\
&+ i\alpha f\frac{e^{\phi/4}}{8}\eta_+ - \alpha\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_+ + \alpha\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_+ \ . \qquad (3.10)
\end{aligned}
$$

3.2 $SU(3)$ structure and tensor decomposition

The existence of the spinor η allows us to define the bilinears

$$J_{mn} := i\eta_-^+\gamma_{mn}\eta_- = -i\eta_+^+\gamma_{mn}\eta_+ \qquad (3.11)$$

$$\Omega_{mnp} := \eta_-^+\gamma_{mnp}\eta_+; \qquad \Omega^*_{mnp} = -\eta_+^+\gamma_{mnp}\eta_- \ . \qquad (3.12)$$

Note that J_{mn} thus defined is real and Ω (Ω^*) is imaginary (anti-) self-dual, as can be seen from (A.8)

$$\Omega_{mnp} = \frac{i}{6}\sqrt{g_6}\ \varepsilon_{mnpijk}\Omega^{ijk} \ . \qquad (3.13)$$

We choose to normalize

$$\eta_+^+\eta_+ = \eta_-^+\eta_- = 1 \ . \qquad (3.14)$$

Using (A.12) one can prove that J, Ω satisfy

$$J_m{}^n J_n{}^p = -\delta_m{}^p \qquad (3.15)$$

$$(\Pi^+)_m{}^n\Omega_{npq} = \Omega_{mpq}; \qquad (\Pi^-)_m{}^n\Omega_{npq} = 0 \ , \qquad (3.16)$$

where

$$(\Pi^\pm)_m{}^n := \frac{1}{2}(\delta_m{}^n \mp iJ_m{}^n) \qquad (3.17)$$

are the projection operators onto the holomorphic/antiholomorphic parts. In other words, J defines an almost complex structure with respect to which Ω is $(3,0)$. Moreover (using (A.12) again) it follows that

$$
\begin{aligned}
\Omega\wedge J &= 0 \\
\Omega\wedge\Omega^* &= \frac{4i}{3}J^3 \ . \qquad (3.18)
\end{aligned}
$$

Therefore J, Ω, completely specify an $SU(3)$ structure on X_6. Some further useful identities are given in appendix B.

In the case of a manifold X_6 of $SU(3)$ structure, the intrinsic torsion decomposes into five modules (torsion classes) $\mathcal{W}_1 \ldots \mathcal{W}_5$. These also appear in the $SU(3)$ decomposition of the exterior derivative of J, Ω. Intuitively this should be clear since the intrinsic torsion parameterizes the failure of the

manifold to be of special holonomy, which can also be thought of as the failure of the closure of J, Ω. More specifically we have

$$\begin{aligned} dJ &= -\frac{3}{2} Im(\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \\ d\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega \; . \end{aligned} \tag{3.19}$$

The classes $\mathcal{W}_1$, $\mathcal{W}_2$ can be decomposed further into real and imaginary parts $\mathcal{W}_1^{\pm}$, $\mathcal{W}_2^{\pm}$.

As a final ingredient before we proceed to the analysis of the next section, we will need the decomposition of the form fields with respect to the reduced structure group $SU(3)$. Using the projectors (3.17) we can decompose the tensors B', H, G in terms of irreducible representations. Explicitly (we henceforth drop the prime on B),

$$B_{mn} = \frac{1}{16}\Omega^*_{mn}{}^s B^{(1,0)}_s + \frac{1}{16}\Omega_{mn}{}^s B^{(0,1)}_s + (\tilde{B}_{mn} + \frac{1}{6} J_{mn} B^{(0)}) \; , \tag{3.20}$$

where $B^{(1,1)}_{mn}$ has been further decomposed into traceless ($\tilde{B}$) and trace ($B^{(0)}$) parts. The normalization above has been chosen so that

$$\begin{aligned} B^{(0)} &= B_{mn} J^{mn} \\ B^{(1,0)}_m &= \Omega_m{}^{np} B_{np} \; . \end{aligned} \tag{3.21}$$

In terms of $SU(3)$ representations we have,

$$B^{(0)} \sim \mathbf{1}; \qquad B^{(1,0)} \sim \mathbf{3}; \qquad B^{(0,1)} \sim \bar{\mathbf{3}}; \qquad \tilde{B} \sim \mathbf{8} \; . \tag{3.22}$$

Note that the tracelessness of $\tilde{B}$ is equivalent to the primitivity condition

$$J \wedge J \wedge \tilde{B} = 0 \; . \tag{3.23}$$

Similarly for the H field we expand,

$$H_{mnp} = \frac{1}{48}\Omega_{mnp} H^{(0)} + (\tilde{H}^{(2,1)}_{mnp} + \frac{3}{4} H^{(1,0)}_{[m} J_{np]}) + \text{c.c.} \; , \tag{3.24}$$

where

$$\begin{aligned} H^{(0)} &= \Omega^{*mnp} H_{mnp} \\ H^{(1,0)}_m &= (\Pi^+)_m{}^s H_{snp} J^{np} \end{aligned} \tag{3.25}$$

and

$$\tilde{H}^{(2,1)} \sim \mathbf{6}; \qquad \tilde{H}^{(1,2)} \sim \bar{\mathbf{6}} \; . \tag{3.26}$$

Finally, for the four-form G we have,

$$G_{mnpq} = \frac{1}{12} G^{(1,0)}_{[m}\Omega^*_{npq]} + \frac{1}{12} G^{(0,1)}_{[m}\Omega_{npq]} + (3\tilde{G}_{[mn}J_{pq]} + \frac{1}{8} G^{(0)} J_{[mn}J_{pq]}) \; , \tag{3.27}$$

where

$$\begin{aligned} G^{(0)} &= G_{mnpq} J^{mn} J^{pq} \\ G^{(1,0)}_m &= \Omega^{npq} G_{mnpq} \\ \tilde{G}_{mn} &= 2(\Pi^+)_m{}^s (\Pi^-)_n{}^t G_{stpq} J^{pq} - \frac{1}{6} J_{mn} G^{(0)} \; . \end{aligned} \tag{3.28}$$

Note that the scalars $B^{(0)}$, $G^{(0)}$ are real whereas $H^{(0)}$ is complex.

4. Analysis

We will start by examining the content of the supersymmetry equations (3.5-3.10), the Bianchi identities (2.2,2.4) and the form equations of motion (2.11,2.12). In section 4.3 we read off the torsion classes of the $SU(3)$ manifold X_6. Some special cases of our solutions are examined in 4.4. Equations (B.11,B.12) of appendix B will be very useful in the following.

4.1 Supersymmetry

We are now ready to analyze the content of equations (3.5-3.10). Our strategy will be to perform all possible contractions with $\eta_\pm^+\gamma^{(n)}$, as is made clear by the following

Lemma: For χ, ϵ constant spinors in $\mathbb{R}^6$, where χ is non-vanishing,

$$\epsilon = 0 \Longleftrightarrow \chi\gamma^{(n)}\epsilon = 0, \quad n = 0, \ldots, 3 \ .$$

Proof: First note that

$$\epsilon^\alpha = 0 \Longleftrightarrow \xi^\alpha C_{\alpha\beta}\epsilon^\beta = 0, \quad \forall\xi \ , \tag{4.1}$$

where ϵ, ξ are spinors in $\mathbb{R}^6$. Clearly, if $\epsilon = 0$ it follows that $\xi^\alpha C_{\alpha\beta}\epsilon^\beta = 0$. Conversely, if $\epsilon \neq 0$ we can assume without loss of generality that $\epsilon^{\alpha=1} \neq 0$ and $C_{\alpha=2\ \beta=1} \neq 0$ (it cannot be that $C_{\alpha 1} = 0, \ \forall\alpha$, as this would imply $det(C) = 0$ whereas C is in fact unitary). It follows that $\xi^\alpha C_{\alpha\beta}\epsilon^\beta \neq 0$, for $\xi^\alpha = \delta^\alpha_2$. Moreover it holds that if χ is a non-vanishing spinor, for any ξ there exist constants $\{\phi^{(n)}_{a_1\ldots a_n}\}$ such that

$$\xi = \sum_{n=0}^{3} \phi^{(n)}_{a_1\ldots a_n}\gamma^{a_1\ldots a_n}\chi \ . \tag{4.2}$$

This follows from the fact that the Gamma matrices $\{\gamma^{(n)}, n = 0, \ldots 3\}$ generate $Gl(4,\mathbb{C})$ which acts transitively on $\mathbb{C}^4 - \{0\} \ni \chi$. For globally-defined spinors on a manifold X_6, as is the case at hand, the above can be generalized to arbitrary points on the tangent bundle. In this case $\{\phi^{(n)}_{a_1\ldots a_n}\}$ become forms on X_6. The lemma follows immediately from (4.1, 4.2).

The 1

We first note that multiplying (3.9) by β (3.10) by α^* adding them together and separating real and imaginary parts, we obtain

$$\begin{aligned} 0 &= (|\alpha|^2 - |\beta|^2)\Big(\frac{5me^{5\phi/4}}{4} - \frac{e^\phi}{64}G^{(0)}\Big) \\ 0 &= mB^{(0)} - \frac{fe^{-\phi/2}}{3} \ , \end{aligned} \tag{4.3}$$

where we noted that $|\alpha|^2 + |\beta|^2 > 0$. We therefore distinguish two cases:

Case 1: $|\alpha| \neq |\beta|$.

The first of equations (4.3) implies

$$G^{(0)} = 80me^\phi \ . \tag{4.4}$$

Substituting this back to (3.9), (3.10) we obtain

$$H^{(0)} = 0 \ . \tag{4.5}$$

Similarly, multiplying (3.7) by β (3.8) by α subtracting one from the other and separating real and imaginary parts, we obtain

$$\begin{aligned} m &= 0 \\ F^{(0)} &= 0 \ , \end{aligned} \tag{4.6}$$

where in the second line we have taken the massless limit (2.6). Finally, plugging the above back to (3.7), (3.8) implies

$$W = 0 \ . \tag{4.7}$$

Hence, this case reduces to compactification to $\mathbb{R}^{1,3}$. This was analyzed in detail in [35], [36], [29] and will not concern us further here.

Case 2: $|\alpha| = |\beta|$.

Without loss of generality, we can choose the phase of the internal spinor η so that

$$\alpha = \beta \neq 0 \ . \tag{4.8}$$

In the following it will be useful to add and subtract equations (3.5,3.6), taking (4.8) into account, to obtain

$$0 = \nabla_m \eta_+ + \frac{1}{2}\partial_m ln|\alpha|^2 \eta_+ + 3if\frac{e^{\phi/4}}{32}\gamma_m \eta_- + \frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np} - 14\delta_m{}^n\gamma^p)\eta_- \tag{4.9}$$

$$\begin{aligned} 0 = &+\frac{1}{2}\partial_m ln\Big(\frac{\alpha}{\alpha^*}\Big)\eta_+ + \frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_+ - \frac{me^{5\phi/4}}{16}\gamma_m\eta_- \\ &+ \frac{e^{\phi/4}}{256}G_{npqr}(\gamma_m{}^{npqr} - \frac{20}{3}\delta_m{}^n\gamma^{pqr})\eta_- \ , \end{aligned} \tag{4.10}$$

In this case the system of equations (3.7-3.10), (4.10) can be solved to give

$$\begin{aligned} mB^{(0)} &= \frac{1}{3}fe^{-\phi/2} \\ H^{(0)} &= \frac{96}{5}me^{7\phi/4} \\ G^{(0)} &= \frac{144}{5}me^{\phi} \\ W &= \Delta\Big(\frac{\alpha}{|\alpha|}\Big)^{-2}(-\frac{1}{5}me^{5\phi/4} + \frac{i}{6}fe^{\phi/4}) \ . \end{aligned} \tag{4.11}$$

The 3

The solution of equations (3.7-3.10) reads

$$\begin{aligned} \partial_m^{(1,0)}\phi &= \frac{3}{8}me^{3\phi/4}B_m^{(1,0)} \\ H_m^{(1,0)} &= 0 \\ G_m^{(1,0)} &= 0 \\ \Delta &= constant \times e^{-\phi/12} \ , \end{aligned} \tag{4.12}$$

where we have defined

$$\partial_m^{(1,0)} := (\Pi^+)_m{}^n \partial_n \ . \tag{4.13}$$

Equation (4.10) then implies

$$Arg(\alpha) = constant \ , \tag{4.14}$$

which, taking (4.11) into account and the fact that W is a constant, implies the following two cases:

$$\begin{aligned} \phi,\ f &= constant \\ B^{(1,0)} &= 0 \end{aligned} \tag{4.15}$$

and $m \neq 0$, or,

$$\begin{aligned} f &= constant \times e^{-\phi/6} \\ Arg(\alpha) &= \frac{\pi}{4} \end{aligned} \tag{4.16}$$

and $m = 0$. The latter case can be seen to reduce to four-dimensional Minkowski space once the Bianchi identities and the equations of motion are imposed, and will not concern us further.

The 6

This representation drops out of equations (3.7-3.10). Equation (4.10) implies

$$\tilde{H}^{(1,2)} = 0 \ . \tag{4.17}$$

The 8

As in the previous case, this representation drops out of equations (3.7-3.10). Equation (4.10) implies

$$\tilde{G} = 0 \ . \tag{4.18}$$

To summarize our results so far, the solution to equations (3.7-3.10,4.10) reads in form notation

$$\boxed{\begin{aligned} mB &= \frac{f}{18} e^{-\phi/2} J + m\tilde{B} \\ H &= \frac{4m}{5} e^{7\phi/4} Re(\Omega) \\ G &= f dVol_4 + \frac{3m}{5} e^{\phi} J \wedge J \\ W &= \Delta \Big(\frac{\alpha}{|\alpha|}\Big)^{-2} \big(-\frac{1}{5} m e^{5\phi/4} + \frac{i}{6} f e^{\phi/4}\big) \\ \phi,\ \Delta,\ f,\ Arg(\alpha) &= constant \ . \end{aligned}} \tag{4.19}$$

In the above we have denoted by $dVol_4$ the volume element of AdS_4 in the warped metric.

The $SU(3)$ structure

Plugging equation (4.9) into the definitions (3.11,3.12) we find

$$\nabla_m J_{kl} = -J_{kl}\partial_m ln|\alpha|^2 + \frac{2}{9} f e^{\phi/4} Re(\Omega_{mkl}) + m e^{3\phi/4} Im(\Omega_{kl}{}^s)\tilde{B}_{ms}$$
$$\nabla_m \Omega_{klt} = -\Omega_{klt}\partial_m ln|\alpha|^2 + 6ime^{3\phi/4} J_{[kl}(\Pi^+)_{t]}{}^s \tilde{B}_{sm} - \frac{4}{3} f e^{\phi/4} J_{[kl}(\Pi^+)_{t]m} \ . \quad (4.20)$$

By antisymmetrizing in all indices we obtain

$$dJ = -J \wedge dln|\alpha|^2 + \frac{2}{3} f e^{\phi/4} Re(\Omega) \quad (4.21)$$

and

$$d\Omega = -\Omega \wedge dln|\alpha|^2 - 2ime^{3\phi/4} J \wedge \tilde{B} - \frac{4i}{9} f e^{\phi/4} J \wedge J \ . \quad (4.22)$$

4.2 Equations-of-motion and Bianchi identities

Taking (4.19) into account, the Bianchi identity (2.4) for the H field implies

$$dRe(\Omega) = 0 \ , \quad (4.23)$$

which is satisfied iff

$$\boxed{|\alpha| = constant \ ,} \quad (4.24)$$

as can be seen from equation (4.22). The Bianchi identity (2.4) for the G field can be seen to be satisfied automatically by noting that $\tilde{B} \wedge \Omega = 0$ and $dJ \wedge J \propto Re(\Omega) \wedge J = 0$. In the latter we have taken (4.21,4.24) into account. Moreover, using (4.19) we see that equation (2.2) implies

$$\boxed{md\tilde{B} = \frac{e^{-\phi/4}}{27}\left(\frac{108m^2}{5}e^{2\phi} - f^2\right)Re(\Omega) \ ,} \quad (4.25)$$

from which it follows that

$$d * \tilde{B} = 0 \ . \quad (4.26)$$

In deriving (4.26) we noted that $J \wedge d\tilde{B} = d(J \wedge \tilde{B}) = d * (\tilde{B} \wedge dVol_4)$, as can be seen from (4.21, B.13). It follows from (4.26) and the Hodge decomposition theorem that $*\tilde{B}$ is harmonic up to an

exact form: $*\tilde{B} = d\chi + Harm$, i.e. $d\tilde{B} = -d * d\chi$, for some globally-defined seven-form χ. We can see that this is indeed the case: using (4.25) it can be shown that

$$
\begin{aligned}
*d * dIm(\Omega) &= \frac{2}{3}e^{\phi/2}(\frac{12m^2}{5}e^{2\phi} - f^2)Im(\Omega) \\
d * d * Re(\Omega) &= \frac{2}{3}e^{\phi/2}(\frac{12m^2}{5}e^{2\phi} - f^2)Re(\Omega)
\end{aligned}
\tag{4.27}
$$

and therefore $d\tilde{B} \propto Re(\Omega) \propto d * d * Re(\Omega) \propto d * dIm(\Omega) \wedge dVol_4$. Similar equations have appeared in the mathematical literature in [16].

Note that (4.24,4.26) are equivalent to the consistency condition $d(d\Omega) = 0$. The corresponding condition for the almost complex structure, $d(dJ) = 0$, is automatically satisfied. However, equation (4.25) is a consequence of supersymmetry and the Bianchi identities, and has to be imposed as an extra condition on the $SU(3)$ structure. We will examine a class of examples with this property in section 5.

Starting from $d(\Omega \wedge \tilde{B}) = 0$, taking (4.22,4.25) into account, one arrives at

$$
\begin{aligned}
m^2|\tilde{B}|^2 &= -\frac{4e^{-\phi}}{27}(\frac{108m^2}{5}e^{2\phi} - f^2) \ , \\
f^2 &\geq \frac{108m^2}{5}e^{2\phi} \ .
\end{aligned}
\tag{4.28}
$$

Using (B.13), it can then be seen that the form equations (2.11,2.12) are satisfied with no further restrictions on the fields. Although not necessary, as was argued in section 2, we have checked that the dilaton equation is also satisfied.

4.3 Torsion classes

Equations (4.21,4.22) can be used to read off the torsion classes of the internal manifold X_6 by comparing with (3.19) (taking (4.24) into account):

$$
\boxed{
\begin{aligned}
\mathcal{W}_1^+ &= 0 \\
\mathcal{W}_1^- &= -\frac{4i}{9}fe^{\phi/4} \\
\mathcal{W}_2^+ &= 0 \\
\mathcal{W}_2^- &= -2ime^{3\phi/4}\tilde{B} \\
\mathcal{W}_3 &= 0 \\
\mathcal{W}_4 &= 0 \\
\mathcal{W}_5 &= 0 \ .
\end{aligned}
}
\tag{4.29}
$$

Note that X_6 belongs to the class of half-flat manifolds. As mentioned in the introduction, the latter are defined by the property $\mathcal{W}_1^+ = \mathcal{W}_2^+ = \mathcal{W}_4 = \mathcal{W}_5 = 0$.

In conclusion, type IIA, $\mathcal{N} = 1$ compactifications on $AdS_4 \times X_6$ are given by (4.19,4.24), where the internal manifold X_6 has $SU(3)$ structure with torsion classes given by (4.29) and $\mathcal{W}_2^-$ is further restricted by (4.25).

4.4 Special cases

In the limit where

$$f^2 = \frac{108m^2}{5} e^{2\phi} \ , \tag{4.30}$$

it follows from (4.28) that $\tilde{B}$ and $\mathcal{W}_2$ vanish. Then all torsion classes vanish except for $\mathcal{W}_1^-$, and X_6 is further restricted to be *nearly-Kähler*. This case was analyzed in detail in [37, 38].

The massless limit should be taken with care, as was explained in section 2. In this case the solution reduces to

$$\begin{aligned}
&F = \frac{1}{9} f e^{-\phi/2} J + \tilde{F} \\
&H = 0 \\
&G = f dVol_4 \\
&W = \frac{i}{6} \Delta \Big(\frac{\alpha}{|\alpha|}\Big)^{-2} f e^{\phi/4} \\
&\phi,\ \Delta,\ f,\ \alpha = constant
\end{aligned} \tag{4.31}$$

with

$$d\tilde{F} = -\frac{2}{27} f^2 e^{-\phi/4} Re(\Omega) \ . \tag{4.32}$$

All torsion classes are zero except for $\mathcal{W}_1^-$, $\mathcal{W}_2^-$ which are given by

$$\begin{aligned}
\mathcal{W}_1^- &= -\frac{4i}{9} f e^{\phi/4} \\
\mathcal{W}_2^- &= -i e^{3\phi/4} \tilde{F} \ .
\end{aligned} \tag{4.33}$$

This solution can be lifted to eleven dimensions, leading to a seven-dimensional internal manifold which is a (twisted) circle fibration over a six-dimensional half-flat base. Compactifications of eleven dimensional supergravity to AdS_4 in which the seven-dimensional internal space is a product of a circle and a six-dimensional nearly Kähler manifold, were considered in [49]. However no well-defined solutions were obtained in this reference. In fact, the results of the present paper imply that no such solutions exist. This can be seen as follows: in order for the manifold X_6 to be nearly Kähler we would have to have $f = 0$. Note that taking the $f \to 0$ limit naively appears to lead to a compactification on $\mathbb{R}^{1,3} \times X_6$ with only the primitive part of the F-flux turned on. However, as we can see from (4.28), F has to vanish and the internal manifold reduces to a Calabi-Yau.

5. Examples

5.1 T^2 over K3

We now construct a class of examples of six dimensional manifolds X_6 with the property that their intrinsic torsion is contained in $\mathcal{W}_1^- \oplus \mathcal{W}_2^-$ and, in addition, the exterior derivative of $\mathcal{W}_2^-$ is proportional to $Re(\Omega^{(3,0)})$ [1]. Our starting point is the work of Goldstein and Prokushkin [50]. These authors have shown that six-dimensional manifolds with $SU(3)$ structure can be constructed as T^2 fibrations over Hermitian four-dimensional manifolds (X_4). The metric on the total space is then of the form

$$g_{base} + (dx + a)^2 + (dy + b)^2 \ , \tag{5.1}$$

where g_{base} is the metric on the base X_4 and a, b are local one-forms on X_4. Moreover a, b satisfy $da = \omega_P$, $db = \omega_Q$, with

$$\frac{[\omega_P]}{2\pi}, \ \frac{[\omega_Q]}{2\pi} \in H^2(X_4, \mathbb{Z}) \ . \tag{5.2}$$

The complex $(3,0)$ form and the almost complex structure on the total space are given by

$$\Omega^{(3,0)} = \{(dx + a) + i(dy + b)\} \wedge \Omega^{(2,0)} \tag{5.3}$$

and

$$J = \omega + (dx + a) \wedge (dy + b) \ , \tag{5.4}$$

where $\Omega^{(2,0)}$, ω are the holomorphic $(2,0)$ form and the hermitian $(1,1)$ form on the base[2] respectively. In the case we are considering X_4 is a Calabi-Yau two-fold (i.e. a K3 surface) and therefore $\Omega^{(2,0)}$, ω are closed.

The two-forms ω_P, ω_Q should have no component in $\Lambda^{0,2}T^*X_4$ in order for the total space X_6 to be complex. However, as was noted in [50], this condition can be relaxed. In fact, for our purposes we will take ω_P, ω_Q to be purely of type $(2,0) \oplus (0,2)$ on the base. Namely we take

$$\begin{aligned} \omega_P &= -\frac{3i}{2}\mathcal{W}_1^- Im(\Omega^{(2,0)}) \\ \omega_Q &= -\frac{3i}{2}\mathcal{W}_1^- Re(\Omega^{(2,0)}) \ , \end{aligned} \tag{5.5}$$

where $\mathcal{W}_1^-$ is an imaginary constant, which should be quantized. We can see this as follows: let $\Gamma_{3,19}$ be the even, self-dual lattice of integral cohomology, where the following identifications are understood,

$$\Gamma_{3,19} \simeq H^2(X_4, \mathbb{Z}) \subset H^2(X_4, \mathbb{R}) \ . \tag{5.6}$$

[1] In this subsection we will write $\Omega^{(3,0)}$ instead of simply Ω, to distinguish from the holomorphic two-form $\Omega^{(2,0)}$ defined in the following.

[2] If the map $\pi : X_6 \mapsto X_6/T^2 \simeq X_4$ defines the fibration, we can extend $\Omega^{(2,0)}$, ω, a, b from X_4 to the total space X_6 by using π^*.

Note that $\Omega^{(2,0)} \in H^2(X_4, \mathbb{C}) \simeq H^2(X_4, \mathbb{R}) \oplus H^2(X_4, \mathbb{R})$. Let us define $x := Re(\Omega^{(2,0)})$, $y := Im(\Omega^{(2,0)})$ so that $x,\ y \in H^2(X_4, \mathbb{R})$. For $u,\ v \in H^2(X_4, \mathbb{R})$ we can define an inner product by

$$u \cdot v := \int_{X_4} u \wedge v \ . \tag{5.7}$$

We have,

$$(x \cdot x + y \cdot y) = \int_{X_4} \Omega^{(2,0)} \wedge \Omega^{(0,2)} = 4Vol(X_4) \ . \tag{5.8}$$

Moreover, it follows from $\Omega^{(2,0)} \wedge \Omega^{(2,0)} = 0$ that $x \cdot y = 0$, $x \cdot x = y \cdot y$, and therefore

$$\frac{1}{Vol(X_4)} x \cdot x = 2 \ . \tag{5.9}$$

I.e. x, y have the same length and are orthogonal to each other. Let Σ be the two-plane defined by x, y. Changing the complex structure on X_4 while keeping $\Gamma_{3,19}$ fixed causes Σ to rotate, spanning the entire space of two-planes in $H^2(X_4, \mathbb{R})$ (see for example [51]). By choosing an appropriate complex structure on X_4, we may arrange so that $x/\sqrt{Vol(X_4)}$, $y/\sqrt{Vol(X_4)} \in \Gamma_{3,19}$, as can be seen from the explicit form of the lattice. It follows from (5.5) that in order for (5.2) to hold, $\mathcal{W}_1^-$ has to be quantized.

From (5.3,5.4,5.5) we can compute the exterior derivatives of $\Omega^{(3,0)}$, J,

$$\begin{aligned} dJ &= \frac{3i}{2} \mathcal{W}_1^- Re(\Omega^{(3,0)}) \\ d\Omega^{(3,0)} &= \frac{3}{2} \mathcal{W}_1^- \Omega^{(2,0)} \wedge \Omega^{(0,2)} \ . \end{aligned} \tag{5.10}$$

Moreover, by noting that $\Omega^{(2,0)} \wedge \Omega^{(0,2)} = 2\omega \wedge \omega$, we can see that the last line can be written as

$$d\Omega^{(3,0)} = \mathcal{W}_1^- J \wedge J + \mathcal{W}_2^- \wedge J \ , \tag{5.11}$$

where

$$\mathcal{W}_2^- := 2\mathcal{W}_1^- \{\omega - 2(dx + a) \wedge (dy + b)\} \ . \tag{5.12}$$

Note that $\mathcal{W}_2^-$ satisfies the primitivity condition $J \wedge J \wedge \mathcal{W}_2^- = 0$. By comparing with (3.19) we conclude that the intrinsic torsion of X_6 is entirely within $\mathcal{W}_1^- \oplus \mathcal{W}_2^-$. In addition, from (5.12) we see that the exterior derivative of $\mathcal{W}_2^-$ is proportional to $Re(\Omega^{(3,0)})$ as promised:

$$d\mathcal{W}_2^- = -6i(\mathcal{W}_1^-)^2 Re(\Omega^{(3,0)}) \ . \tag{5.13}$$

However as we can see from (4.25,4.29), there are no values of f, m, other than $f = m = 0$, that can fit with the above equation. In other words, although the example examined in this section satisfies (1.1, 1.2), it does not correctly reproduce the proportionality constant in equation (4.25) except for the rather special case where all fluxes vanish and the internal manifold reduces to a Calabi-Yau threefold.

5.2 Iwasawa manifold

The following example, given in [15], of dynamic half-flat $SU(3)$ structure is based on the Iwasawa manifold $\mathcal{M}$. Consider the following basis of one-forms on $\mathcal{M}$:

$$\begin{aligned} de^5 &= -e^{14} - e^{23} \\ de^6 &= -e^{13} - e^{42} \\ de^i &= 0, \qquad\qquad\qquad i = 1,2,3,4 \ , \end{aligned} \tag{5.14}$$

where we use the notation $e^{ij} := e^i \wedge e^j$. Then for all $t \in \mathbb{R}^+$ the following defines a half-flat $SU(3)$ structure

$$\begin{aligned} M^2 J &= t^2(e^{12} + e^{34}) + t^{-2} e^{56} \\ M^3 \Omega &= t(e^1 + ie^2) \wedge (e^3 + ie^4) \wedge (e^5 + ie^6) \ , \end{aligned} \tag{5.15}$$

compatible with the metric

$$M^2 g = t^2 \sum_{i=1}^{4} e^i \otimes e^i + t^{-2} \sum_{i=5}^{6} e^i \otimes e^i \ . \tag{5.16}$$

We have introduced a mass scale M so that the einbeine e^i are dimensionless. It is then straightforward verify that

$$\begin{aligned} dJ &= \frac{3i}{2} \mathcal{W}_1^- Re(\Omega) \\ d\Omega &= \ \mathcal{W}_1^- J \wedge J + \mathcal{W}_2^- \wedge J, \end{aligned} \tag{5.17}$$

where

$$\begin{aligned} M^{-1}\mathcal{W}_1^- &:= -\frac{2i}{3} t^{-3} \\ M\mathcal{W}_2^- &:= \frac{8i}{3}(-\frac{1}{2}t^{-1}e^{12} - \frac{1}{2}t^{-1}e^{34} + t^{-5}e^{56}) \ . \end{aligned} \tag{5.18}$$

By comparing with (3.19) we see that the intrinsic torsion is contained in $\mathcal{W}_1^- \oplus \mathcal{W}_2^-$. Note that $\mathcal{W}_2^-$ satisfies the primitivity condition $\mathcal{W}_2^- \wedge J \wedge J = 0$, as it should. Moreover we find

$$d\mathcal{W}_2^- = -6i(\mathcal{W}_1^-)^2 Re(\Omega) \ , \tag{5.19}$$

which is the same as equation (5.13) of the previous example! However, this is not a coincidence as the example based on the Iwasawa manifold is in fact a special case of the T^2 over $K3$ fibration considered in 5.1. This can be seen as follows: equations (5.14) allow us to express the vielbeine in terms of coordinates x^i such that [3]

$$\begin{aligned} e^5 &= dx^5 - x^1 dx^4 + x^3 dx^2 \\ e^6 &= dx^6 - x^1 dx^3 - x^4 dx^2 \\ e^i &= dx^i, \qquad\qquad\qquad i = 1,2,3,4 \ . \end{aligned} \tag{5.20}$$

[3] In the following we set $t,\ M = 1$ for simplicity.

The coordinates x^i, $i = 1 \dots 4$, parameterize a T^4 base on which we can define a hermitian (in fact Kähler) $(1,1)$ form and a holomorphic (2,0) form in analogy with the previous section:

$$\begin{aligned} \omega &= \frac{i}{2}(dz^1 \wedge d\bar{z}^1 + dz^2 \wedge d\bar{z}^2) \\ \Omega^{(2,0)} &= dz^1 \wedge dz^2 \ , \end{aligned} \tag{5.21}$$

where $dz^1 := dx^1 + idx^2$, $dz^2 := dx^3 + idx^4$. Equation (5.16) can be written as

$$g = \sum_{i=1}^{4} (dx^i)^2 + (dx^5 + a)^2 + (dx^6 + b)^2 \ , \tag{5.22}$$

where $a := -x^1 dx^4 + x^3 dx^2$, $b := -x^1 dx^3 - x^4 dx^2$. I.e. the metric is of the form (5.1) and, moreover, it can be seen that (5.5, 5.13) are satisfied for $\mathcal{W}_1^- = -2i/3$. In other words, the example based on the Iwasawa manifold is a degenerate instance of the T^2 over $K3$ fibration presented in 5.1, whereby the $K3$ base is replaced by a T^4.

Acknowledgments

We would like to thank Claus Jeschek for useful discussions.

A. Spinor conventions

In all dimensions the Gamma matrices are taken to obey

$$(\Gamma^M)^+ = \Gamma^0 \Gamma^M \Gamma^0 \ , \tag{A.1}$$

where the Minkowski metric is mostly plus. Antisymmetric products of Gamma matrices are defined by

$$\Gamma^{(n)}_{M_1 \dots M_n} := \Gamma_{[M_1} \dots \Gamma_{M_n]} \ . \tag{A.2}$$

A.1 Spinors in $D = 6$

The charge conjugation matrix in six Euclidean dimensions satisfies

$$C^{Tr} = C; \qquad (C\gamma^m)^{Tr} = -C\gamma^m \ . \tag{A.3}$$

The fundamental (4-dimensional, chiral) spinor representation η_+ is complex and we define η_- by

$$\eta_+^+ = \eta_-^{Tr} C \ , \tag{A.4}$$

which also implies

$$\eta_-^+ = \eta_+^{Tr} C \ . \tag{A.5}$$

A useful formula which follows from the above is

$$\left(\gamma^{(n)} \eta_\pm\right)^* = (-)^n C \gamma^{(n)} \eta_\mp \ . \tag{A.6}$$

The chirality matrix defined by

$$\gamma_7 := -i\gamma_1 \dots \gamma_6; \qquad \gamma_7^2 = 1 \ , \tag{A.7}$$

can be used to express the Hodge-dual of an antisymmetric product of gamma-matrices

$$i\gamma^{(n)} = (-)^{\frac{1}{2}k(k-1)} * \gamma^{(D-n)}\gamma_7 \ . \tag{A.8}$$

Fierz rearrangement follows from

$$\chi_\pm^\alpha \psi^\beta = \frac{1}{4}\phi^\pm (P_\pm C^{-1})^{\alpha\beta} - \frac{1}{4}\phi_m^\pm (P_\pm \gamma^m C^{-1})^{\alpha\beta} + \frac{1}{8}\phi_{mn}^\pm (P_\pm \gamma^{mn} C^{-1})^{\alpha\beta} - \frac{1}{48}\phi_{mnp}^\pm (P_\pm \gamma^{mnp} C^{-1})^{\alpha\beta} \ , \tag{A.9}$$

where

$$\phi_{m_1 \dots m_k} := \chi_\pm^{Tr} C \gamma_{m_1 \dots m_k} \psi \tag{A.10}$$

and

$$P_\pm := \frac{1}{2}(1 \pm \gamma_7) \ . \tag{A.11}$$

Note that ϕ^+_{mnp} (ϕ^-_{mnp}) is imaginary (anti-) self-dual, as follows from (A.8). In particular, using definitions (3.11,3.12) we find

$$\begin{aligned} \eta_-^\alpha \eta_+^\beta &= \frac{1}{4}(P_- C^{-1})^{\alpha\beta} + \frac{i}{8} J_{mn} (P_- \gamma^{mn} C^{-1})^{\alpha\beta} \\ \eta_+^\alpha \eta_+^\beta &= -\frac{1}{48}\Omega_{mnp}(P_+ \gamma^{mnp} C^{-1})^{\alpha\beta} \\ \eta_-^\alpha \eta_-^\beta &= \frac{1}{48}\Omega^*_{mnp}(P_- \gamma^{mnp} C^{-1})^{\alpha\beta} \ . \end{aligned} \tag{A.12}$$

A.2 Spinors in $D = 1 + 3$

The charge conjugation matrix in $1 + 3$ dimensions satisfies

$$C^{Tr} = -C; \qquad (C\gamma^\mu)^{Tr} = -C\gamma^\mu \ . \tag{A.13}$$

The fundamental (2-dimensional, chiral) spinor representation θ_+ is complex and we define θ_- by

$$\overline{\theta}_+ = \theta_-^{Tr} C \ , \tag{A.14}$$

which also implies

$$\overline{\theta}_- = -\theta_+^{Tr} C \ , \tag{A.15}$$

where

$$\overline{\theta} := \theta^+ \gamma^0 \ . \tag{A.16}$$

A useful formula which follows from the above is

$$\left(\gamma^{(n)}\theta_\pm\right)^* = \pm(-)^n C\gamma^0\gamma^{(n)}\theta_\mp \ . \tag{A.17}$$

The chirality matrix is defined by

$$\gamma_5 := i\gamma_0 \dots \gamma_3; \qquad \gamma_5^2 = 1 \ . \tag{A.18}$$

A.3 Spinors in $D = 10 \to 4 + 6$

The charge conjugation matrix in $1 + 9$ dimensions satisfies

$$C^{Tr} = -C; \qquad (C\Gamma^M)^{Tr} = C\Gamma^M \ . \tag{A.19}$$

The fundamental (16-dimensional, chiral) spinor representation $\epsilon_\pm$ is real and we define the reality condition by

$$\bar{\epsilon}_\pm = \epsilon_\pm^{Tr} C \ . \tag{A.20}$$

The chirality matrix is defined by

$$\Gamma_{11} := \Gamma_0 \dots \Gamma_9; \qquad \Gamma_{11}^2 = 1 \ . \tag{A.21}$$

We decompose the ten-dimensional Gamma matrices as

$$\begin{aligned} \Gamma^\mu &= \gamma^\mu \otimes 1, \qquad & \mu &= 0, \dots 3 \\ \Gamma^m &= \gamma_5 \otimes \gamma^m, \qquad & m &= 4 \dots 9 \ . \end{aligned} \tag{A.22}$$

It follows that

$$C_{10} = C_4 \gamma_5 \otimes C_6; \qquad \Gamma_{11} = \gamma_5 \otimes \gamma_7 \ . \tag{A.23}$$

B. $SU(3)$ structure

Using (A.12) one can show the following useful identities satisfied by J, Ω:

$$\Omega_{[abc}\Omega^*_{def]} = \frac{2i}{5}\varepsilon_{abcdef} \tag{B.1}$$

$$\Omega_{abc}\Omega^{*def} = 48(\Pi^+)_{[a}{}^{[d}(\Pi^+)_b{}^e(\Pi^+)_{c]}{}^{f]} \tag{B.2}$$

$$\Omega_{abc}\Omega^{*ade} = 16(\Pi^+)_{[b}{}^{[d}(\Pi^+)_{c]}{}^{e]} \tag{B.3}$$

$$\Omega_{abc}\Omega^{*abd} = 16(\Pi^+)_c{}^d \tag{B.4}$$

$$|\Omega|^2 = 48 \tag{B.5}$$

$$\varepsilon_{abcdef}J^{cd}J^{ef} = -8J_{ab} \tag{B.6}$$

$$\varepsilon_{abcdef}J^{ef} = -6J_{[ab}J_{cd]} \tag{B.7}$$

$$\varepsilon_{abcdef} = -15J_{[ab}J_{cd}J_{ef]} \tag{B.8}$$

Note that from the last one it follows that

$$dVol_6 = -\frac{1}{6}J^3 \ . \tag{B.9}$$

The following relations are useful in analyzing the supersymmetry conditions of section 4.1.

$$
\begin{aligned}
0 &= (\Pi^+)_m{}^n \gamma_n \eta_- \\
\gamma_{mn} &= i J_{mn} \eta_+ + \frac{1}{2} \Omega_{mnp} \gamma^p \eta_- \\
\gamma_{mnp} \eta_- &= -3i J_{[mn} \gamma_{p]} \eta_- - \Omega^*_{mnp} \eta_+ \ .
\end{aligned} \tag{B.10}
$$

The above equations together with tensor decompositions (3.20,3.24,3.27) give

$$
\begin{aligned}
B_{np}(\gamma_m{}^{np} - 14\delta_m{}^n \gamma^p)\eta_- &= \Big(\frac{5i}{3} B^{(0)} \gamma_{mt} - 16\tilde{B}_{mt} - \frac{3}{4} \Omega_{mt}{}^s B_S^{(0,1)}\Big)\gamma^t \eta_- - B_m^{(0,1)} \eta_+ \\
H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n \gamma^{pq}) &= \Big(- H^{(0)*} g_{mt} - \frac{9}{2} \Omega_t{}^{pq} \tilde{H}^{(1,2)}_{mpq} - \frac{3}{2} \Omega_m{}^{pq} \tilde{H}^{(1,2)}_{tpq} + \frac{3i}{2} \Omega_{mt}{}^p H_p^{(0,1)}\Big)\gamma^t \eta_- \\
&\quad - (12 i H_m^{(1,0)} + 6 i H_m^{(0,1)})\eta_+ \\
G_{npqr}(\gamma_m{}^{npqr} - \frac{20}{3} \delta_m{}^n \gamma^{pqr}) &= \Big(\frac{7}{3} G^{(0)} g_{mt} + 32 i \tilde{G}_{mt} - \frac{1}{3} \Omega_{mt}{}^p G_p^{(0,1)}\Big)\gamma^t \eta_- + \frac{20}{3} G_m^{(0,1)} \eta_+
\end{aligned} \tag{B.11}
$$

and

$$
\begin{aligned}
\gamma^{mn} B_{mn} \eta_- &= -i B^{(0)} \eta_- - \frac{1}{2} B_t^{(0,1)} \gamma^t \eta_+ \\
\gamma^{mnp} H_{mnp} \eta_+ &= H^{(0)*} \eta_- + 3 i H_t^{(0,1)} \gamma^t \eta_+ \\
\gamma^{mnpq} G_{mnpq} \eta_+ &= -3 G^{(0)} \eta_- - 2 G_t^{(0,1)} \gamma^t \eta_+ \ .
\end{aligned} \tag{B.12}
$$

Finally, one can show the following Hodge-dualizations

$$
\begin{aligned}
*J &= \frac{1}{2} J \wedge J \wedge dVol_4 \\
*(J \wedge J) &= 2 J \wedge dVol_4 \\
*\Omega &= i \Omega \wedge dVol_4 \\
*\tilde{B} &= -J \wedge \tilde{B} \wedge dVol_4 \ ,
\end{aligned} \tag{B.13}
$$

where in proving the last one we used the fact that $\tilde{B}$ is primitive.

References

[1] A. Strominger, "Superstrings With Torsion," Nucl. Phys. B **274** (1986) 253.

[2] B. de Wit, D. J. Smit and N. D. Hari Dass, "Residual Supersymmetry Of Compactified D = 10 Supergravity," Nucl. Phys. B **283** (1987) 165.

[3] J. Polchinski and A. Strominger, "New Vacua for Type II String Theory," Phys. Lett. B **388** (1996) 736, hep-th/9510227.

[4] K. Becker and M. Becker, "M-Theory on Eight-Manifolds," Nucl. Phys. B **477** (1996) 155, hep-th/9605053.

[5] J. Michelson, "Compactifications of type IIB strings to four dimensions with non-trivial classical potential," Nucl. Phys. B **495** (1997) 127, hep-th/9610151.

[6] T. R. Taylor and C. Vafa, "RR flux on Calabi-Yau and partial supersymmetry breaking," Phys. Lett. B **474** (2000) 130, hep-th/9912152.

[7] P. Mayr, "On supersymmetry breaking in string theory and its realization in brane worlds," Nucl. Phys. B **593** (2001) 99, hep-th/0003198.

[8] G. Curio, A. Klemm, D. Lüst and S. Theisen, "On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes," Nucl. Phys. B **609** (2001) 3, hep-th/0012213.

[9] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66** (2002) 106006, hep-th/0105097.

[10] O. DeWolfe and S. B. Giddings, "Scales and hierarchies in warped compactifications and brane worlds," Phys. Rev. D **67** (2003) 066008, hep-th/0208123.

[11] C. Angelantonj, S. Ferrara and M. Trigiante, "New D = 4 gauged supergravities from N = 4 orientifolds with fluxes," JHEP **0310** (2003) 015, hep-th/0306185.

[12] C. Angelantonj, R. D'Auria, S. Ferrara and M. Trigiante, "K3 x T**2/Z(2) orientifolds with fluxes, open string moduli and critical points," Phys. Lett. B **583** (2004) 331, hep-th/0312019.

[13] A. Giryavets, S. Kachru, P. K. Tripathy and S. P. Trivedi, "Flux compactifications on Calabi-Yau threefolds," JHEP **0404** (2004) 003, hep-th/0312104.

[14] S. Kachru and A. K. Kashani-Poor, "Moduli potentials in type IIA compactifications with RR and NS flux," hep-th/0411279.

[15] S. Chiossi and S. Salamon, "The intrinsic torsion of $SU(3)$ and G_2 structures," math.DG/0202282.

[16] S. Chiossi and A. Swann, "G_2-structures with torsion from half-integrable nilmanifolds," math.DG/0404554.

[17] S. Gurrieri, J. Louis, A. Micu and D. Waldram, "Mirror symmetry in generalized Calabi-Yau compactifications," Nucl. Phys. B **654** (2003) 61, hep-th/0211102.

[18] S. Gurrieri and A. Micu, "Type IIB theory on half-flat manifolds," Class. Quant. Grav. **20** (2003) 2181, hep-th/0212278.

[19] J. P. Gauntlett, D. Martelli, S. Pakis and D. Waldram, "G-structures and wrapped NS5-branes," Commun. Math. Phys. **247** (2004) 421, hep-th/0205050.

[20] P. Kaste, R. Minasian, M. Petrini and A. Tomasiello, "Kaluza-Klein bundles and manifolds of exceptional holonomy," JHEP **0209** (2002) 033, hep-th/0206213.

[21] G. L. Cardoso, G. Curio, G. Dall'Agata, D. Lüst, P. Manousselis and G. Zoupanos, "Non-Kaehler string backgrounds and their five torsion classes," Nucl. Phys. B **652** (2003) 5, hep-th/0211118.

[22] J. P. Gauntlett and S. Pakis, "The geometry of D = 11 Killing spinors," JHEP **0304** (2003) 039, hep-th/0212008.

[23] P. Kaste, R. Minasian, M. Petrini and A. Tomasiello, "Nontrivial RR two-form field strength and SU(3)-structure," Fortsch. Phys. **51** (2003) 764, hep-th/0301063.

[24] K. Behrndt and C. Jeschek, "Fluxes in M-theory on 7-manifolds and G structures," JHEP **0304** (2003) 002, hep-th/0302047.

[25] J. P. Gauntlett, D. Martelli and D. Waldram, "Superstrings with intrinsic torsion," Phys. Rev. D **69** (2004) 086002, hep-th/0302158.

[26] P. Kaste, R. Minasian and A. Tomasiello, “Supersymmetric M-theory compactifications with fluxes on seven-manifolds and G-structures,” JHEP **0307** (2003) 004, hep-th/0303127.

[27] J. P. Gauntlett and J. B. Gutowski, “All supersymmetric solutions of minimal gauged supergravity in five dimensions,” Phys. Rev. D **68** (2003) 105009, Erratum-ibid. D **70** (2004) 089901, hep-th/0304064.

[28] S. Fidanza, R. Minasian and A. Tomasiello, “Mirror symmetric SU(3)-structure manifolds with NS fluxes,” hep-th/0311122.

[29] G. Dall'Agata and N. Prezas, “N = 1 geometries for M-theory and type IIA strings with fluxes,” Phys. Rev. D **69** (2004) 066004, hep-th/0311146.

[30] K. Behrndt and C. Jeschek, “Fluxes in M-theory on 7-manifolds: G2, SU(3) and SU(2) structures,” hep-th/0406138.

[31] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, “Supersymmetric AdS(5) solutions of M-theory,” Class. Quant. Grav. **21** (2004) 4335, hep-th/0402153.

[32] T. House and A. Micu, “M-theory compactifications on manifolds with G(2) structure,” hep-th/0412006.

[33] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, “Supersymmetric AdS backgrounds in string and M-theory,” hep-th/0411194.

[34] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, “The Baryonic Branch of Klebanov-Strassler Solution: a Supersymmetric Family of SU(3) Structure Backgrounds,” hep-th/0412187.

[35] M. Grana, R. Minasian, M. Petrini and A. Tomasiello, “Supersymmetric backgrounds from generalized Calabi-Yau manifolds,” JHEP **0408** (2004) 046, hep-th/0406137.

[36] M. Grana, R. Minasian, M. Petrini and A. Tomasiello, “Type II strings and generalized Calabi-Yau manifolds,” hep-th/0409176.

[37] K. Behrndt and M. Cvetic, “General N = 1 supersymmetric flux vacua of (massive) type IIA string theory,” hep-th/0403049.

[38] K. Behrndt and M. Cvetic, “General N = 1 supersymmetric fluxes in massive type IIA string theory,” hep-th/0407263.

[39] J. H. Schwarz, “Superconformal Chern-Simons theories,” JHEP **0411** (2004) 078, hep-th/0411077.

[40] C. Nunez, I. Y. Park, M. Schvellinger and T. A. Tran, “Supergravity duals of gauge theories from F(4) gauged supergravity in six dimensions,” JHEP **0104** (2001) 025, hep-th/0103080.

[41] E. Abbena, S. Garbiero and S. Salamon, “Almost hermitian geometry on six dimensional nilmanifolds,” math.DG/0007066.

[42] S. Chiossi and A. Fino, “Conformally parallel G_2 structures on a class of solvmanifolds,” math.DG/0409137.

[43] I. A. Bandos, J. A. de Azcarraga, J. M. Izquierdo, M. Picon and O. Varela, “On BPS preons, generalized holonomies and D = 11 supergravities,” Phys. Rev. D **69** (2004) 105010, hep-th/0312266.

[44] L. J. Romans, “Massive N=2a Supergravity In Ten-Dimensions,” Phys. Lett. B **169** (1986) 374.

[45] F. Giani and M. Pernici, “N=2 Supergravity In Ten-Dimensions,” Phys. Rev. D **30** (1984) 325.

[46] I. C. G. Campbell and P. C. West, “N=2 D = 10 Nonchiral Supergravity And Its Spontaneous Compactification,” Nucl. Phys. B **243** (1984) 112.

[47] M. Huq and M. A. Namazie, “Kaluza-Klein Supergravity In Ten-Dimensions,” Class. Quant. Grav. **2** (1985) 293 [Erratum-ibid. **2** (1985) 597].

[48] U. Gran, “GAMMA: A Mathematica package for performing Gamma-matrix algebra and Fierz transformations in arbitrary dimensions,” hep-th/0105086.

[49] A. Lukas and P. M. Saffin, “M-theory compactification, fluxes and AdS(4),” hep-th/0403235.

[50] E. Goldstein and S. Prokushkin, “Geometric model for complex non-Kaehler manifolds with SU(3) structure,” Commun. Math. Phys. **251** (2004) 65, hep-th/0212307.

[51] P. S. Aspinwall, “K3 surfaces and string duality,” hep-th/9611137.